BODY WAVES IN INHOMOGENEOUS MEDIA*

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Several investigators have demonstrated that in anisotropic media there are three types of waves. For a long time, I have suspected that it is also possible to demonstrate the existence of three types of propagation in inhomogeneous media which are isotropic. I have not been able to do this in general, but only in certain special cases which, in themselves, have no seismological interest. Obviously, if three wave types exist in these special cases, then they must also be considered in the more general cases. If the elastic parameters vary only in the X direction the equations of motion can be written in the form

$$(\lambda + \mu) \frac{\partial \theta}{\partial X} + \mu \nabla^2 U + \theta \frac{\partial \lambda}{\partial X} + 2 \frac{\partial \mu}{\partial X} \frac{\partial U}{\partial X} = \rho \frac{\partial^2 U}{\partial T^2} \qquad (1)$$

$$(\lambda + \mu) \frac{\partial \theta}{\partial Y} + \mu \nabla^2 V + \left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y}\right) \frac{\partial \mu}{\partial X} = \rho \frac{\partial^2 V}{\partial T^2} \qquad (2)$$

$$(\lambda + \mu) \frac{\partial \theta}{\partial Z} + \mu \nabla^2 W + \left(\frac{\partial U}{\partial Z} + \frac{\partial W}{\partial X}\right) \frac{\partial \mu}{\partial X} = \rho \frac{\partial^2 W}{\partial T^2} \qquad (3)$$

The quantities λ and μ are the Lame coefficients; ρ is the density; θ is the dilatation; and U, V, and W are the X, Y, and Z components of displacement.

Independent of the physical hypotheses, it is always possible to split the displacement field into curl free and divergence free parts. Consider first the special case in which the Lame coefficients vary linearly with X and the density is constant. In order to obtain an equation in the dilatation only, it was necessary to require that the curl vanish. The resulting equation has the form

$$(\lambda + 2\mu) \nabla^2 \theta + 2 \frac{\partial \theta}{\partial X} \frac{\partial}{\partial X} (\lambda + 2\mu) = \rho \frac{\partial^2 \theta}{\partial T^2} \qquad (4)$$

On the other hand, if the dilatation is set equal to zero, (1) reduces to an equation in the X component of displacement and (2) and (3) can be combined in such a way that they yield an equation for the X component of curl (χ). The resulting equations have the form

$$\mu \nabla^2 U + 2 \frac{\partial \mu}{\partial X} \frac{\partial U}{\partial X} = \rho \frac{\partial^2 U}{\partial T^2} \qquad (5)$$

$$\mu \nabla^2 \chi + \frac{\partial \mu}{\partial X} \frac{\partial \chi}{\partial X} = \rho \frac{\partial^2 \chi}{\partial T^2} \qquad (6)$$

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(4), (5), and (6) have the same form in the differential coefficients and differ only in the constant coefficients. In the homogeneous case, (5) and (6) become identical. It seems likely that one of these equations describes the SV and the other the SH.

I have also considered the special case in which the Lame coefficients and the density vary according to the law $\lambda = lf(x)$, $\mu = mf(x)$, $\rho = rf(x)$. Provided that the ratio $f'(x)/f(x)$ is constant, it is possible to obtain equations which are identical in form to (4), (5), and (6). Therefore, we have two special cases in which it is possible to demonstrate the existence of three differential equations with three different sets of constant parameters. The point which I wish to emphasize is that each of these equations describes a type of motion.

DISCUSSION

C. H. Dix: The characteristics of equations (5) and (6) are the same. This means that the velocities are the same. The terms in $\partial\mu/\partial X$ do not affect the velocity of the first arrival.

C. Eckart: There are two ray theories. The theory of characteristics rigorously takes account of the first arrival. The Fresnel, Hamilton, de Broglie ray theory applies to the lines of flow of the energy. The flow of the energy in the later phases does depend on the terms in $\partial\mu/\partial X$.